Math 53, Discussions 116 and 118 **2022-09-14**

Working with vectors

Questions

Question 1. Do the vectors $\langle 1, 2, 4\rangle$ and $\langle -2, -2, 2\rangle$ form an acute, right, or obtuse angle (when placed tail to tail)?

Question 2. If $\mathbf{u}$ and $\mathbf{v}$ are two vectors in $\mathbb{R}^3$, we have the two identities

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos\theta, \qquad |\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| \sin\theta$$

where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$. Which of these formulas is better for determining θ, and why?

Question 3. Find where the line through the points $(1, 2, 3)$ and $(2, 4, 6)$ intersects the plane $x + 3y + z = 20$.

Question 4. On a previous discussion worksheet, you investigated the equation

$$(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$$

where $\mathbf{r} = \langle x, y\rangle$, $\mathbf{a} = \langle a_1, a_2\rangle$ and $\mathbf{b} = \langle b_1, b_2\rangle$ where a_1, a_2, b_1, b_2 are constants. After some tedious algebra, we found that this equation describes a circle such that the line segment connecting (a_1, a_2) and (b_1, b_2) is its diameter.

With your newfound knowledge (geometric understanding of the dot product), see if you can arrive at that same conclusion geometrically.